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Full Length Research Paper

Garched investment decision making with real risk

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Abstract
Actual future market risks (systematic or non-diversifiable) of investment portfolios are determined in this paper. Future returns are first forecasted using past returns and GARCH (General Autoregressive Conditional Heteroskedastic) models. A Real Risk Weighted Pricing Model (RRWPM) is used to estimate future systematic risk among other parameters and determines the future costs of the portfolios. Forecasted random error is then calculated as a random variable and used to determine probability density estimates of portfolios market risk. This enables future actual market risks of portfolio investments to be derived hence facilitating proper future investment decision making.

Keywords: Market risk; GARCH; Probability Density Estimates; Random Error.
JEL Classification: G32

1.0 Introduction

Recent years have seen a surge of interest in econometric models of changing conditional variance. Probably the most widely used but by no means the only such models, are the family of ARCH (Autoregressive Conditional Heteroskedastic) models introduced by Engle (1982). Researchers have fruitfully applied the new ARCH methodology in asset pricing models. For example, Engle and Bollerslev (1986) used GARCH (1,1) to model the risk premium on the foreign exchange market and Bollerslev *et al* (1988) extended GARCH (1,1) to a multivariate context to test a conditional CAPM (Capital Asset Pricing Model) with time varying covariance. However their results show that shocks may persist in one norm and die out in another, so the conditional moments of GARCH (1, 1) may explode even when the process itself is strictly stationary and ergodic Nelson (1990). Achia *et al* (2008) revealed that the GARCH (1, 1) model provided a good explanation of the dynamics of the market returns but failed to obey the efficient market principle indicating that there is market risk.

This paper uses a RRWPM as determined by Anyika *et al* (2010a) that avoids the explosion of conditional moments of GARCH (1, 1). With this model the relationship between the actual and estimated values with GARCH forecasted time series data is almost perfect. With the determination of total forecasted risk using the RRWPM the assumption of an efficient market need not be upheld. Section 2 outlines how returns of a portfolio of stocks are forecasted using the GARCH (1, 1) model, section 3 uses forecasted returns in section 2 and RRWPM to determine forecasted future cost and total risk. Section 4 calculates estimates of white noise using an estimator derived by Anyika *et al* (2010b) and determines probability density estimates of the portfolio systematic risk using the Gaussian kernel, section 5 surveys the process of using the forecasted returns with the RRWPM to result into future real cost of capital and other parameters. Probability estimates of future portfolio risks are estimated as well as actual market risk. Finally section 6 summarizes what has been done and concluded based on the results.

2.0 Forecasting Returns using Garch (1, 1)

ARCH models make the conditional variance of the time t prediction error a function of time system parameters, exogenous and lagged endogenous variables, and past prediction errors.

For each integer t, let ξ_t be a model's (scalar) prediction error, b a vector of parameters, x_t a vector of predetermined

variables, and σ_t^2 the variance of ξ_t given information at time t ,
A univariate ARCH model based on Engel sets

$$\xi_t = \sigma_t z_t \quad (1)$$

Where, z_t □ $i.i.d$, with $E(z_t) = 0$, $\mathrm{var}(z_t) = 1$,

$$\sigma_t^2 = \sigma^2(\xi_{t-1}, \xi_{t-2}, ..., t, x_t, b) = \sigma^2(\sigma_{t-1}z_{t-1}, \sigma_{t-2}z_{t-2}, ..., t, x_t, b) \quad (2)$$

Equation (1) - (2) can be given a multivariate interpretation as suggested by B rooks *et al* (2003), in which case z_t is a n by one vector and σ_t^2 is an n by n matrix. We refer to any model of the form of equation (1) - (2) whether univariate or multivariate, as an ARCH model. The most widely used specification of equation (2) are the linear ARCH and GARCH models introduced by Engle and Bollerslev respectively, which make σ_t^2 linear in lagged values of $\xi_t^2 = \sigma_t^2 z_t^2$, by defining

$$\sigma_t^2 = \omega + \sum_{j=1}^{p} \alpha_j z_{t-j}^2 \sigma_{t-j}^2 \quad (3)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{p} \alpha_j z_{t-j}^2 \sigma_{t-j}^2 \quad (4)$$

respectively, where ω , α_j , and β_i are non negative. Since equation 3 is a special case of equation 4 we refer to both equations 3 and 4 as GARCH models, to distinguish them as special cases of equation 2. The GARCH – M model of Engle and Bollerslev adds another equation

$$R_t = a + b\sigma_t^2 + \xi_t \quad (5)$$

in which σ_t^2 , the conditional variance of R_t, enters the conditional mean of R_t as well. For example if R_t is the return on a portfolio at time t , its required rate of return may be linear in its risks as measured by σ_t^2 . Therefore the GARCH – M model is used for forecasting in this research.

3.0 Garched-Real Risk Weighted Price Model

The RRWPM is used to determine the forecasted cost of equity and real risk thus the model will be called Garched – Real Risk Weighted Price Model (G – RRWPM).
To determine the G - RRWPM we let the forecasted weighted expected returns be

$$E\left(R_{l_{wg}}\right) = a_{l_{wg}} + b_{l_{wg}} E\left(R_{m_g}\right) \quad (6)$$

Where, $a_{l_{wg}} = \sum_{i=1}^{\infty} w_i a_i$, $b_{l_{wg}} = \sum_{i=1}^{\infty} w_i b_i$, w_i is the weight of forecasted security i , a is the constant return unique to security i, b_i is a measure of the sensitivity of the return of security i to the return on the market index, $E\left(R_{l_{wg}}\right)$ is the weighted expected return of forecasted security i , $E\left(R_{mg}\right)$ is the weighted expected return of forecasted market index.

Then take weighted forecasted diversifiable risk to be

$$\sigma_{h_{wg}} = \left(c_{l_{wg}} + d_{l_{wg}}\right)^{1/2} \quad (7)$$

and weighted forecasted non – diversifiable risk as

$$\sigma_{G_{wg}} = \left(c_{l_{wg}} + e_{1g}\right)^{1/2} \quad (8)$$

Where, $c_{l_{wg}} = \sum_{i=1}^{\infty} w_i^2 \sigma_i^2$, $d_{l_{wg}} = 2\sum_{i=1}^{\infty}\sum_{\substack{i=1 \\ i \neq j}}^{\infty} w_i w_j \sigma_{ij}$, $e_{1g} = \sum_{i=1}^{\infty} \sigma_{e_i}^2$, σ_j^2 is the variance of the forecasted market index, σ_i^2 variance of security i , $\sigma_{e_i}^2$ variance of random error of security i .
To find the weight of investment i that will maximize expected returns and minimize total variance we apply the classical optimization method with no constraints as given by Rao (1994) . We thus differentiate the expression;

$$E\left(R_{l_{wg}}\right) - c_{l_{wg}} - d_{l_{wg}} = a_{l_{wg}} + b_{l_{wg}} E\left(R_{mg}\right) - c_{l_{wg}} - d_{l_{wg}} \quad (9)$$

With respect to w_i , and differentiate

$$2c_{l_{wg}} + d_{l_{wg}} + e_{1g} \quad (10)$$

With respect to w_i , where $E\left(R_{l_{wg}}\right) - c_{l_{wg}} - d_{l_{wg}}$ are maximum returns (derived by subtracting diversifiable portfolio variance from portfolio expected returns), and $2c_{l_{wg}} + d_{l_{wg}} + e_{1g}$ is the total variance (derived by adding portfolio variance to non-diversifiable variance)
Note: The second derivative of the differential in 10 is equal to $-2\sum_{i=1}^{\infty} \sigma_i^2$ implying that w_i obtained will always maximize returns and that in 11 is equal to $4\sum_{i=1}^{\infty} \sigma_i^2$ implying that w_i obtained will always minimize risk.
Equate the differentials in 9 to 10 to get the value of w_i ,

$$a_{l_{wg}} + E(R_{mg})b_l - 2c_{l_{wg}} - 2d_{l_{wg}} = 2c_{l_{wg}} + 2d_{l_{wg}} + 2c_{l_{wg}} \quad (11)$$

$$-6c_{l_{wg}} = 4d_{l_{wg}} - a_{l_{wg}} - E(R_{mg})b_l$$

w_j is similarly derived.

Thus $$w_j = -\frac{2\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} x_i \sigma_{ij}}{3\sum_{j=1}^{\infty} \sigma_j^2}$$

Replacing it in expression 11 gives the value of

$$w_i = \frac{3\sum_{j=1}^{\infty}\sigma_j\left(\sum_{i=1}^{\infty}a_i + E\left(R_{mg}\right)\sum_{i=1}^{\infty}b_i\right)}{18\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sigma_i^2\sigma_j^2 - 8\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sigma_{ij}^2} \quad (12)$$

For $i = 1$

$$w_1 = \frac{3\sum_{j=1}^{\infty}\sigma_j\left(\sum_{i=1}^{\infty}a_1 + E\left(R_{mg}\right)\sum_{i=1}^{\infty}b_1\right)}{18\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sigma_1^2\sigma_j^2 - 8\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sigma_{1j}^2} \quad (13)$$

Once these weights are determined, they are substituted in equation 6 to give maximum returns and in both equations 7 and 8 to give minimum total risk. The costs of capital are also determined which enable accurate future predictions.

4.0 Determining Forecasted White Noise

The non-diversifiable variance estimator

$$\sigma_n^2 = \sum_{i=1}^{\infty} x_i\sigma_i^2 + \sum_{i=1}^{\infty}\sigma_{e_i}^2 \quad (14)$$

as derived by Anyika *et al* (2005) indicates the presence of random error in the risk estimator. This error is taken to be white noise (wn) thus it can be said to be a random variable $V_1, V_2, V_3, \ldots, V_\infty$ which is mutually independent and identically distributed. This is estimated from a sample of data by first varying the variance of individual return values of r_i such that:

$$w\hat{n}_i = \Psi \sum_{i=1}^{\infty}\left\{\sigma^2_{r_i}\sigma^2_n\right\} \quad (15)$$

Where $\Psi = \frac{z-2}{(z-1)^2}$, z is the total number of returns and (15) is the predicted random error.

From (15) the actual value of $w\hat{n}_i$ is given by

$$wn_i = \sum_{i=1}^{\infty} x_i^2\sigma_i^2\Theta^2 + \wp \quad (16)$$

Where Θ^2 and $\wp$ are values representing the scale (mean) and location (variance) parameters. These parameters are determined such that the bias and variance of the actual and predicted values of wn are minimized as follows;

$$\mathrm{var}(w\hat{n}_i, wn_i) =$$

$$\frac{2}{z-1}w\hat{n}_i^2 - \left(\frac{2}{z-1}\right)^2 w\hat{n}_i wn_i + \frac{2}{z-1}wn_i^2 \quad (17)$$

The values of Θ and $\wp$ which will minimize variance are given by the partial derivatives of Θ and $\wp$, f_Θ and $f_\wp$ respectively. After several iterations;

$$f_\Theta = 2wn_i - \frac{2}{z-1}w\hat{n}i = \wp$$

$$f_\wp = \frac{2}{z-1}w\hat{n}_i - wn_i = \wp$$

White noise of the real risk is given by the equation

$$wn_i = \frac{4}{3(z-1)}w\hat{n}_i \quad (18)$$

As determined by Anyika *et al* (2010b) where $\Psi \sum_{i=1}^{\infty}\left\{\sigma^2_{r_i}\sigma_{G_w}\right\}$ are forecasted stock returns and $\Psi = \frac{z-2}{(z-1)^2}$

The white noise is estimated as a random error from its definition of being mutually independent identically distributed random variable with constant mean and variance and $\mathrm{cov}(wn_i, wn_{i+l}) = 0$

Where $l = \pm1, \pm2, \cdots$

It is generally known that the value of the bandwidth is of critical importance while the shape of the kernel function has little practical impact.

The value of the bandwidth that minimizes the AMISE is given by

$$h_{AMISE} = \left[R(k)/\mu_2(k)^2 R(f'')\right]^{1/5} n^{-1/5}$$

The Gaussian kernel by Sheather and Jones (1991) is used to determine the probability estimates.

It is given by $K(v) = \frac{1}{\sqrt{2\pi}}\exp\left(\frac{-y^2}{2}\right)$

5.0 Results

5.1. *Preliminary Data*

Twenty stock portfolios were picked randomly from the New York Stock Exchange. The New York Share Index (NYSE) is used as the market share index and the long term Treasury bond as the risk free asset. The monthly returns of the twenty stocks, the NYSE and Treasury bond since July 1996 - September 2009 were forecasted for different periods and their returns calculated. A sample of the forecasted parameters and returns of Toyota using Matlab forecasting software are given by table 1

5.1.1 Surveys

The forecasted returns are substituted into equation 6 to give the forecasted real risk weighed expected returns, cost of equity, and the total real risk as shown in the table 3. Non – diversifiable risk estimated using equation 9 is used to calculate white noise as an independent random variable as given by equation 14 and presented in table 4. Gaussian kernel is used to determine the probability estimates of non- diversifiable risk using white noise as an independent random variable and thus calculate actual non- diversifiable risk as tabulated in Table 5.

6.0 Conclusion

The r_2 value for RRWPM averages 0.999 for the twenty forecasted stocks indicating that it is almost a perfect estimator of cost of equity. This is in comparison to the CAPM model which averages 0.25. This shows that a RRWPM avoids the explosion of conditional moments of GARCH (1, 1) since this has not deterred the RRWAM to be a perfect estimator of cost of equity. The actual non-diversifiable risk determined using derived white noise enables one make future predictions on the various portfolios. If we compare the market risk 12 months after the credit crunch in the United States of America (US) economy as shown in table 7 and that at the height of the crunch as shown in table 6 we see that 12 months later the risks are much lower as it is true with the US economy right now. In particular the companies which needed financial assistance to stay afloat 12 months ago AIG, TM and FORD, had market risks of, 4390.919, 237.5173 and 954.7601 respectively and 12 months later they have market risks of 82.54386, 10.46597 and 34.72481 respectively. Thus this research is a true reflection of the actual market risks. Also the least risky stocks currently (twelve months later) include BPH, TIF, AMC and VICL. This is a good prediction in relation to other methods like Value at risk, Capital Asset Pricing Model since it is in comparison with other Portfolios.

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Appendices

Table 1: Conditional Probability Distribution: Gaussian Parameter for Toyota Forecasts.

Parameter	Standard Value	T Error	Statistic
C	0.007386	0.0067064	1.1013
MA(1)	-0.05125	0.1055	-0.4858
K	0.00019654	0.00028766	0.6833
GARCH(1)	0.90643	0.063142	14.3555
ARCH(1)	0.034543	0.027396	1.2609
Leverage(1)	0.084469	0.063075	1.3392

Where: Parameter refers to the Standard value, the T error and the Statistic value.
Standard value = the determined values of the unknowns.
T Error = the error values in determining the standard values.
Statistic = the ratio of Standard value to T Error,
C and K are the constant values used in estimating the MA (1), GARCH (1) and ARCH (1),
Leverage (1) = the value that compares the actual value and estimated value.

Table 2: 18 Month Forecasts of Toyota Returns

0.0851	0.0877	0.0899
0.0856	0.0881	0.0902
0.0861	0.0885	0.0906
0.0865	0.0888	0.0909
0.0869	0.0892	0.0912
0.0873	0.0895	0.0915

These values are determined using the standard values plus the previous error term.

Table 3: A Table of Values for the Survey of RRWPM with 12 Month Forecasted Returns

COMPANY	BETA	ALPHA	r_2	s_{ey}	$E(r_i)$	σ_n	w_i
TM	-9.54292	1.00134	0.99999	86.6696	1.152211	41.2738	210876
HMC	3.431529	1.01662	0.99994	30.2805	0.88325	17.2158	17913.27
PARD	-8.24994	0.99774	0.99741	75.8045	3.63853	24.3518	5274.66
VICL	-9.29791	4.56046	0.99969	114.899	2.5129	14.2396	1303.08
DWCH	-3.13457	1.02443	0.9968	38.65	1.98147	13.9194	2926.67
BP	2.041693	1.00334	0.99999	110.8701	0.90452	36.738	90836.6
STI	17.49379	1.00795	0.99999	172.977	2.10794	69.8255	185721
PNC	8.196005	1.00763	0.99001	180.9757	1.54059	29.1505	9829.81
AIG	115.6898	1.00203	0.99999	871.5171	4.29225	316.416	1538914
F	-3.34379	0.99638	1.00000	177.2241	2.39588	118.724	1378801
AMR	1.4558	0.96866	0.99979	41.4946	2.89675	25.7771	11355.9
BPH	-0.53771	1.000191	0.99999	0.007131	1.42972	0.537676	20.4607
CTL	-1.75989	0.98106	0.99935	85.5529	1.0979	31.5394	34748.1
PFE	2.22284	1.02424	0.99984	32.7984	0.987518	18.4911	15985.3
RTI	-1.97823	0.99658	0.99896	132.1698	1.74984	29.7557	33276.7
GSK	8.56784	1.00197	0.99999	76.18854	0.72527	32.6397	176875
BCE	-0.18985	0.99658	0.99454	58.8545	1.36037	23.6328	10082.1
SBGI	3.819447	1.01005	0.99999	74.32407	2.851	44.0047	77721.6
YAH	-6.02508	0.94946	0.99423	50.99261	2.24331	21	5798.92
TIF	-0.00427	0.73454	0.99898	0.093718	2.2033	0.033178	0.579569

Where: TM = Toyota Motors, HMC = Honda motors, PARD = Ponard pharmaceuticals, VICL = Vical, DWCH = Data watch, BP = British power, STI = Sun Trust Bank, PNC = PNC Finance services, AIG = American International group, F = Ford, AMR = Amr company BHP = BHP Billiton, CTL = CENTURY TEL, PFE = Pfizer, RTI = RTI Intl Metals, GSK = GlaxoSmithKline, BCE = BCE Company, SBGI = Sinclair Broadcast Group, YH = Yahoo group, TF = Tiffany.

r_2 = The coefficient of determination, s_{ey} = The standard error for the y estimate and $E(r_i)$ = Cost of equity.

Table 4: A Table of Estimated White Noise of Forecasted Returns

COMPANY	TM	HMC	PARD	VICL	DWCH	BP
wn_i	0.000068	0.000052	0.001436	0.000436	0.001299	0.00004
STI	PNC	AIG	F	AMR	BPH	CTL
0.000217	0.00101	0.000108	0.000282	0.000458	0.000096	0.000076
PFE	RTI	GSK	BCE	SBGI	YAH	TIF
0.000055	0.000197	0.000033	0.000084	0.000408	0.000022	0.000195

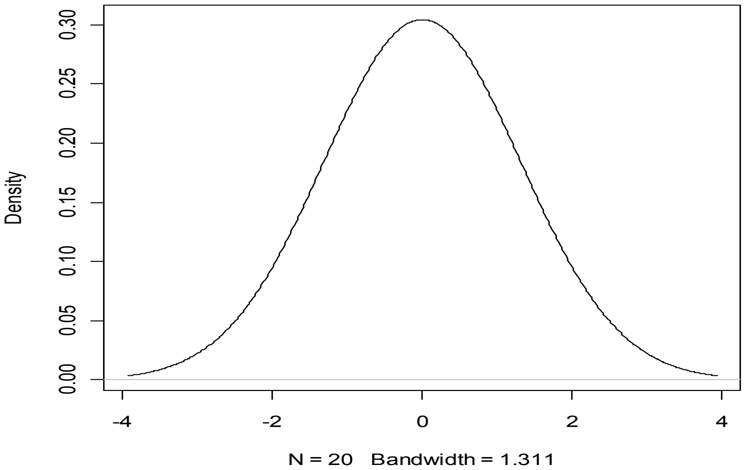
White noise in table 4 is determined using equation18 on page 8

Table 5: A Summary of the Results of a Gaussian Kernel Density Estimation of Forecasted White Noise

X	y
Min. :-3.9316733	Min. :0.003405
1st Qu.:-1.9654271	1st Qu.:0.024243
Median:0.0008191	Median:0.098588
Mean : 0.0008191	Mean:0.126682
3rd Qu.: 1.9670653	3rd Qu.:0.229129
Max . : 3.9333115	Max. :0.304343

This table show data divided into four quarters

Figure I: A Plot of the Density Estimates of a Gaussian Kernel Density Estimation of Forecasted White Noise



N represents the total number of companies being investigated

Table 6: Final Results of a Gaussian Kernel Density Estimation of Forecasted White Noise and Actual Non-Diversifiable Risk

Company	F	Probability Estimates	σ_n *
TM	-0.62972	0.253574	10.46597
HMC	-0.66799	0.250696	4.315931
PARD	2.675201	0.560296	13.64422
VICL	0.259601	0.323869	4.611757
DWCH	2.344264	0.560296	7.456496
BP	-0.69795	0.323869	9.127267
STI	-0.26942	0.535691	19.59819
PNC	1.646155	0.248442	13.52311
AIG	-0.53272	0.280674	82.54386
F	-0.1124	0.463907	34.72481
AMR	0.312744	0.260872	8.451437
BPH	-0.56243	0.292484	0.139062
CTL	-0.61065	0.327866	8.04281
PFE	-0.66142	0.258635	4.644776
RTI	-0.31773	0.255009	8.243527
GSK	-0.71329	0.25119	8.070741
BCE	-0.58966	0.27704	6.063864
SBGI	0.191005	0.247267	14.0247
YAH	-0.74105	0.245201	5.149221
TIF	-0.32256	0.276677	0.00918

* Is calculated by multiplying the probability estimates with estimate real non-diversifiable risk.

TABLE 7: Final Results of White Noise and Kernel Density Estimation of Portfolios of Stocks

	wn	F	Probabilities	Actual σ_n
YH	0.000729	0.827722	0.63566	18.64391
TIF	0.00027	-0.27869	0.491	31.1785
TM	0.00011	-0.66196	0.4414	237.5173
HM	0.000128	-0.62098	0.4467	12.61034
PONARD	0.001551	2.809139	0.979	27.38263
VIC	0.00046	0.179302	0.5511	0.309222
DAWT	0.001456	2.580143	0.9388	24.0145
BP	0.000113	-0.65714	0.442	46.4984
SUNTB	0.00011	-0.66437	0.441	47.7603
PNC	0.000142	-0.58723	0.4511	3.552864
AIG	0.000657	0.654167	0.613	4390.919
FORD	0.000308	-0.18709	0.5033	954.7601

AMR	0.000491	0.254027	0.521	13.11357
BPH	0.000227	-0.38234	0.4778	0.837106
CTL	0.0000884	-0.71655	0.4342	2.408507
PFE	0.0000988	-0.69146	0.4375	20.37875
RTI	0.000238	-0.35582	0.4813	4.35769
GSK	0.0000872	-0.71933	0.4339	15.53796
BCE	0.00022	-0.39921	0.4756	99.44796
STGI	0.000227	-0.38234	0.4778	7.601798

The last two values in Table 6 are the first two in Table 7.